Performance Analysis of Coherent DS-CDMA in Nakagami Multipath Fading Channel
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ABSTRACT

The coherent reception of Direct Sequence Code Division Multiple Access(DS-CDMA) signals in a multipath fading channel is considered. The channel model assumes independent paths with Nakagami fading statistics. This model includes the Rayleigh channel as a special case and a reasonable model for a frequency selective fading channel. The bit error rate(BER) performance of a RAKE receiver under various multipath fading conditions is derived and evaluated. The results indicate that over 50 users may access the channel simultaneously under typical fading conditions.

1. INTRODUCTION

Spread spectrum techniques are well known and have been used successfully in military communication systems for several decades. Certain advantages, such as multipath mitigation and interference suppression, make them a feasible alternative to existing multiuser mobile communication systems[1]. Spread spectrum multiple access systems are known to support more users than conventional time division multiple access(TDMA) and frequency division multiple access(FDMA) systems. Direct –sequence spread spectrum(DSSS) is usually preferred over other spread spectrum techniques because of its low implementation cost and because coherent demodulation can be used in these systems. Asynchronous DSSS is usually employed for the mobile environment so that no network synchronization is necessary. The CDMA system is a multi-access interference- limited system, and it performance degrades as the number of users increases [2]. Furthermore, the introduction of additional users into these systems only gives rise to very graceful performance degradation [3]. The bit error rate(BER) performance of each user in a CDMA system is also affected by the fading statistics of the received signal. A series of propagation experiments conducted in typical urban/
suburban areas have revealed the multipath nature of the mobile radio channel resulting from reflections, refractions and scattering by buildings and other obstructions in the vicinity of the mobile[4]. Multipath propagation can be resolved at the receiver of a Direct Sequence Code-Division multiple-access (DS-CDMA) system that uses a bandwidth much larger than the coherence bandwidth of the channel. In this paper, a channel model incorporating Nakagami multipath fading is developed and the BER performance of a DS-CDMA system operating through it using coherent demodulation is analyzed. For the particular cases of interest (m=1 and m=0.75), the fading statistics do not contain a seculars component in order to demodulate coherently, the implicit assumption is made that some type of carrier recovery and fade mitigation technique is used. One method has been found effective in combining multipath Rayleigh fading. Another method to obtain the coherent reference is via channel parameter estimation circuits [5]. The receiver considered is a RAKE receiver with a variable number of taps to allow for varying degrees of diversity.

2. DS CDMA SYSTEM MODEL

A Transmitter

Let \( a_k(t) \) denote the code sequence waveform of the k-th user, and let \( \{a^{(k)}_j\} \) be the corresponding sequence of elements of \{+1, -1\}. Then

\[
a_k(t) = \sum_{j=-\infty}^{\infty} a^{(k)}_j P_c(t - jT_c),
\]

Where \( P_c(T) = 1 \) for \( 0 \leq t \leq T_c \) and equals zero otherwise. Similarly, the data signal waveform may be written as

\[
b_k(t) = \sum_{j=-\infty}^{\infty} b^{(k)}_j P(t - jT).
\]

The transmitted signal for the k-th user is, therefore,

\[
s_k(t) = \text{Re}[S_k(t)e^{j\omega_0 t}],
\]

Where

\[
S_k(t) = \sqrt{2P}a_k(t)b_k(t)e^{j\theta_k}
\]

And where \( P \) is the average transmitted power, common to all users, \( \omega_0 \) is the common carrier frequency, and \( \theta_k \) is the phase of the k-th carrier. Assuming asynchronous operation, each non-reference user signal is misaligned relative to the reference signal by an amount \( \tau_k \), \( k=1,2…K \); thus, the composite signal at the input to the channel is

\[
s_T(t) = \text{Re}[S_T(t)e^{j\omega_0 t}], \quad \text{……………… (1)}
\]

Where

\[
S_T(t) = \sum_{k=0}^{K} \sqrt{2P}a_k(t - \tau_k)b_k(t - \tau_k)e^{j\theta_k},
\]
A commonly used model for a frequency selective multipath channel is a finite-length tapped delay line as shown in figure 1 for the $k$-th user, where the $L_p^{(k)}$ tap weights $\{\alpha_i^{(k)}\}$ are independent Nakagami random variables with probability density function (pdf)

$$p(\alpha_i^{(k)}) = M(\alpha_i^{(k)}, M, \Omega_i^{(k)}),$$

$$M(R, m, \Omega) = \frac{2^{m R_m R_m^2 - 1}}{\Gamma(m) \Omega^m} e^{-(\frac{m}{\Omega}) R_m^2},$$

Fig. 1. Multipath Channel Model.
and the phases \( \{\psi_{i}^{(k)}\} \) are iid random variables uniform in \((0, 2\Pi)\) and are independent of \( \{\alpha_{i}^{(k)}\} \). In equation 2, \( m \) is a fade parameter assumed common for all paths and is equal to the inverse of the ‘normalized variance’ of \( (\alpha_{i}^{(k)})^2 \), i.e.,

\[
m = E^2[(\alpha_{i}^{(k)})^2] / \text{var}[(\alpha_{i}^{(k)})^2];
\]

For this reason, \( 1/m \) is termed the ‘fading figure’; \( m \) is also related to the commonly used scintillation index, \( S_k \), by \( m = (S_k)^2 \). The parameter \( \Omega_{i}^{(k)} \) is the second moment of \( \alpha_{i}^{(k)} \) (i.e., \( \Omega_{i}^{(k)} = E[(\alpha_{i}^{(k)})^2] \)), and is assumed to be related to the second moment of the initial path strength \( \Omega_{0}^{(k)} \) by

\[
\Omega_{i}^{(k)} = \Omega_{0}^{(k)} e^{-\delta t}, \delta \geq 0.
\]

This functional form for \( \Omega_{i}^{(k)} \) accounts for the decay of average path strength as a function of path delay; the parameter \( \delta \) reflects the rate at which this decay occurs. The shape of the decay function in equation 3 is sometimes referred to as the multipath intensity profile (MIP). The MIP is assumed to be exponential here, and actual measurements indicate that this is fairly accurate for a congested urban area.

\( N(t) \) in fig 1 is the complex valued low-pass-equivalent AWGN with two-sided spectral density \( \eta_0 \); therefore, \( N(t) \) is a circularly symmetric, zero-mean gaussian random process with covariance function \( E[N(t)N^*(\tau)] = 2\eta_0 \delta(t-\tau) \). The number \( L_{p}^{(k)} \) is related to the maximum delay spread, \( \Delta \), of the channel and is assumed to be less than \( N \); since \( \Delta \) is typically less than \( 5\mu s \), this is a reasonable assumption for bit rates of 200KHz or less.

C Receiver Model

The receiver structure is shown in the Figure 2 for the reference user \((k=0)\), where the superscript has been omitted for convenience. This receiver is essentially a coherent RAKE receiver[6], where the number of taps, \( L_k \), is a variable parameter less than or equals to \( L_p \). The matched filter is matched to the reference user’s CDMA code and is assumed to have achieved time synchronization with the initial path of the reference signal. Because the period of the spreading sequence is larger than the bit interval, the matched filter subsequence needs to be updated every \( T \) seconds.
The tap weights and phases are assumed to be perfect estimates of the channel parameters. In practice, these estimates may be obtained from separate circuits[5], and then fed to the demodulator; or the estimation and coherent demodulation may be done jointly through tone calibration techniques. The sampling times of the receiver are \( nT + (L_R - 1)T_c \), where \( n \) is an integer index. For example, to detect the zeroth information bit \( b_0 \), the sample time would be \( T + (L_R - 1)T_c \); the ‘T’ term is from the usual matched filter sampling time and the \( (L_R - 1)T_c \) term is from the \( (L_R - 1) \) paths following the initial path.

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**3. PERFORMANCE ANALYSIS**

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term is from the usual matched filter sampling time and the ‘$(L_R-1)T_c$’ term is from the
$(L_R-1)$ paths following the initial path. With the input to the channel as given in equation 1.
And nothing that the channel responds to each user independently, the output of the channel
(which is also the input to the receiver) may be written as (in the sequel, only the complex
envelope will be given)

$$R_x(t) = \sum_{k=0}^{K} R_k(t) + N(t),$$

Where

$$R_k(t) = \sqrt{2P} \sum_{n=0}^{L_M-1} a_k(t-nT_c-\tau_k)b_k(t-nT_c-\tau_k)\alpha_n^{(k)} e^{i\gamma_n^{(k)}},$$

And where \(\{\gamma_n^{(k)}\} = \{\gamma_n^{(k)} + \phi_k\}\) are iid and uniformly distributed in \((0,2\pi)\).

The response of the reference receiver to \(R_T(t)\) at any sampling instant, say at \(t=T+(L_R-1)T_c\), may be separated into a signal component, denoted by \(U_s\), and three noise
components, denoted by \(U_{mp}, U_{ma}\), and \(U_N\); the first noise component is due to ‘self-noise’
from the multipath waveform of the reference signal, the second component is due to
‘multiple access’ noise from all the other users, and the third component is due to AWGN.
Thus the total response is

$$U = U_s + U_{mp} + U_{ma} + U_N.$$  

Since both the noise and fading processes are assumed stationary, it is clear that the error
probability at only one sampling instant need be evaluated; moreover, as usual in BER
calculations, the path strengths and phases are assumed constant during any bit interval.

**A. conditional Error Probability**

The bit error probability, conditioned on \(\{\alpha_n\}\) and \(\alpha_0(t)\), is calculated. All variance
calculations, unless stated otherwise, are also conditioned on both \(\{\alpha_n\}\) and \(\alpha_0(t)\).
From the channel and receiver models, \( U_N \) may be represented as

\[
U_N = \sqrt{2P} \sum_{n=0}^{L-1} \alpha_n e^{-j\eta_n} \int_{nT_c}^{T_n+nT_c} a_0(\tau - nT_c)N(\tau)d\tau.
\]

Letting \( t = \tau - nT_c \) in the above integral yields the more convenient expression

\[
U_N = \sqrt{2P} \sum_{n=0}^{L-1} \alpha_n e^{-j\eta_n} \int_{0}^{T} a_0(t)N(t+nT_c)dt.
\]

Conditioned on \( \{\alpha_n\} \) and \( a_0(t) \), \( U_N \) is a conditional complex Gaussian random variable with zero mean and conditional variance

\[
E[U_NU_N^* \mid \{\alpha_n\}, a_0(t)] = 2P \sum_{n=0}^{L-1} \alpha_n^2 (2\eta_0T).
\]

since the conditional statistics of \( U_N \) do not depend on \( a_0(t) \), the conditioning on \( a_0(t) \) may be removed without destroying the Gaussian nature of \( U_N \); the conditioning on \( \{\alpha_n\} \) is retained. Also the conditioned on \( \{\alpha_n\}_{n=0}^{L-1} \), each term in the above sum is zero-mean and circularly symmetric (\( N(T) \) being zero-mean and circularly symmetric implies that the integrals are also zero-mean and circularly symmetric, the exponential factor merely adds an independent random phase to each term and does not affect its statistics). Therefore, \( U_N \) is gaussian, zero-mean and circularly symmetric; furthermore, the terms are pair wise uncorrelated, and thus

\[
\sigma_N^2 = \text{var}[U_N \mid \{\alpha_n\}] = E[U_NU_N^* \mid \{\alpha_n\}]
\]

\[
= 4E\eta_0 \sum_{n=0}^{L-1} \alpha_n^2, \quad \text{............... (4)}
\]

Where \( E = PT \) is the average transmitted energy-per-bit.

To determine \( U_{ma} \), consider the receiver response to the \( k \)-th (\( k \neq 0 \)) signal, conditioned on \( a_0(t) \) and \( \{\alpha_n\} \),

\[
U_{ma}^{(k)} = \sqrt{2P} \sum_{n=0}^{L-1} \alpha_n e^{j\eta_n} G_k(T+nT_c).
\]

Where \( G_k(.) \) is the matched filter response to \( R_k(.) \); denoting the impulse response of the matched filter as \( h(\tau) \), the response to \( R_k(.) \) Changing the dummy
variable of integration to $t = \tau - nT$, the filter response may now be written as If the period of the CDMA signature sequence is large relative to the processing gain, then the sequences may

$$G_k(T + nT) = \sqrt{2P} \sum_{i=1}^{L_k-1} \alpha_i e^{j2\pi f\tau} \int_{-\frac{T}{2}}^{\frac{T}{2}} q_i(t) q_i(t - (n-1)T - \tau_k) e^{j2\pi f(t - (n-1)T - \tau_k)} \, dt.$$  

be modeled as random binary sequences, where each chip in a sequence is independently determined, and the sequences are mutually independent; using this model for the code sequences, the integral inequation5. call it $I_{n.t}^{(k)}(\tau_k)$, can be shown to be a zero-mean random variable with conditional variance

$$\frac{2T^2}{3N} + \frac{T^2}{6N} \sum_{i=1}^{N-2} a_i a_{i+1}. \quad \text{note that the second term of this conditional variance is proportional to}$$

the $(N-1)$ chip) partial auto correlation of the spreading sequence, and therefore its magnitude is small relative to the first term for almost all possible sequences. In fact, even though the signature sequences are commonly modeled as being randomly generated, in practice, they are PN sequences that have small partial auto correlations by design[7]. In light of the relative insignificance of the second term, only the dominant first term will be retained in the subsequent analysis. Thus from equation 5. $G_k(T + nT)$ is taken to be zero-mean with conditional variance

$$\sigma^2 = 2P \left( \frac{2T^2}{3N} \right) \sum_{i=0}^{L_k-1} E[(\alpha_i^{(k)})^2]$$

$$= 2P \left( \frac{2T^2}{3N} \right) \sum_{i=0}^{L_k-1} \Omega_i^{(k)} = 2P \left( \frac{2T^2}{3N} \right) \Omega_0^{(k)} q(L_p^{(k)}, \delta),$$

Furthermore, conditioned on $a_0(t)$ and $\{\alpha_n\}$, the $U_{ma}^{(k)}$ are independent for different k, as may be seen by writing $U_{ma}^{(k)}$ as

$$U_{ma}^{(k)} = 2P \sum_{m=0}^{L_m-1} \alpha_n \sum_{i=0}^{L_k-1} \alpha_i^{(k)} e^{j2\pi f(t_i - \tau_k)} I_{m,n}^{(k)}(\tau_k).$$

Where the $I_{n,t}^{(k)}(\tau_k)$ are independent for different k because the spreading
wave forms \( \{a_k(t)\} \), as well as the data wave forms \( \{b_k(t)\} \), are independent for different \( k \); the \( \{\alpha_i^{(k)}\} \) are independent by assumption, and the \( \{\xi^{(k)}_n - \psi_n\} \) are independent because the \( \xi^{(k)}_n \) and \( \psi_n \) are iid and uniform in \( (0,2\pi) \) [8]. Also, \( U_{ma}^{(k)} \) has bounded absolute third moments since \( I_{n-1}(\tau_k) \) is bounded by \( T \) and \( \{\alpha_i^{(k)}\} \) has finite absolute moments of all finite order. With these properties of \( U_{ma}^{(k)} \), Liapounoff’s version of the central-limit-theorem [9] is applicable to the sequence \( U_{ma}^{(k)} \), and therefore as the number of users \( K \to \infty, U_{ma} = \sum_{k=1}^{K} U_{ma}^{(k)} \) is asymptotically conditionally normal with zero-mean and conditional variance

\[
\sigma_{ma}^2 = \left( \frac{8E^2}{3N} \right) \sum_{k=1}^{K} \Omega_0^{(k)} q(L_p^{(k)}, \delta) \sum_{n=0}^{L_p-1} \alpha_n^2.
\]

The use of Gaussian approximation in BER calculations is well documented and it has been shown to be quite accurate even for small values of \( K(<10) \) when the BER is \( 10^{-3} \) or greater.The receiver is in time synchronism with the initial path of \( R_0(t) \), and that the sampling time is \( T + L_K -1)T_c \), the response to \( R_0(t) \), which by definition is equal to \( U_s + U_{mp} \) may be written as

\[
U_s + U_{mp} = 2pT_c x^T A y,
\]

\[
U_s = b_0 2E \sum_{n=0}^{L_p-1} \alpha_n^2.
\]

Note that conditioned on \( \{\alpha_n\}_{n=0}^{L_p-1} \) and \( b_0 \), \( U_s \) is a constant. First, the multipath noise may be considered as the \( (K+1) \)-st multiple access user, where instead of \( L_p \) paths, there would be only \( L_p -1 \) paths at the input to the receiver because one path goes to \( U_i \). Second, because the multipath noise is chip synchronized with the receiver, a factor of \( 3/2 \) is introduced in \( \sigma_{mp}^2 \). Therefore combining these two relationships between \( U_{ma} \) and \( U_{mp} \), \( \sigma_{mp}^2 \) is approximated as

\[
\sigma_{mp}^2 \approx 4 \frac{E^2}{N} \Omega_0 q(L_p, \delta) - 1 \sum_{n=0}^{L_p-1} \alpha_n^2.
\]

The decision variable \( \text{Re} [U] \) is a Gaussian random variable with mean \( U_s \) and variance

\[
\sigma_T^2 = \frac{1}{2} \left[ \sigma_{n}^2 + \sigma_{mp}^2 + \sigma_{ma}^2 \right].
\]

From symmetry, the conditional error probability does not depend on the data bit; therefore assume \( b_0 = 1 \), denote
PERFORMANCE ANALYSIS

\[ S = \frac{1}{\Omega_0} \sum_{n=0}^{l_n-1} \alpha^2_n \quad \text{---------6} \]

\[ \gamma = \left[ \frac{q(L_p, \delta) - 1}{2N} + \frac{Kq(L_p, \delta)}{3N} + \frac{\eta_0}{2E\Omega_0} \right]^{-1} \quad \text{---------7} \]

When \( L_p = 1 \) (no multipath), then \( q(L_p, \delta) = 1 \) and the first term of equation\( 7. \) vanishes; if \( \Omega_0 = 1 \) as well (no fading), then \( \gamma \) reduces to the SNR expression derived by Pursley for a one path channel.

B. Average Error probability

The average bit error probability is calculated from the conditional error probability by averaging \( P_b(S) \) with the pdf of \( S, \ P_b(S) \). Since, from Eqs. 6 and 3, \( S \) is a sum of squares of Nakagami random variables with densities \( M(\alpha, m, e^{-\delta_i}). \) \( S \) may be approximated as the square of another Nakagami random variable whose density is given by

\[ P(\sqrt{S}) \approx M(\sqrt{S}, m_s, \Omega_s), \text{where} \]

\[ m_s = \left( \sum_{i=0}^{l_n-1} e^{-\delta_i} \right)^2 \quad \sum_{i=0}^{l_n-1} \frac{(e^{-\delta_i})^2}{m} = m \frac{q(L_p, \delta)^2}{q(L_p, 2\delta)} \]

The average bit error probability \( P_b, \) then, may be written as

\[ P_b = \int_0^\infty P_b(S)P(\sqrt{S})d\sqrt{S} = \int_0^\infty \varphi(-\sqrt{7S})M(\sqrt{S}, m_s, \Omega_s)d\sqrt{S} \quad \text{---------8} \]

The integral in equation\( 8. \) is the usual formulation for the bit error probability of coherent binary signaling in a Nakagami channel and has a known solution,

\[ P_b = \frac{\gamma_s}{\sqrt{1 + \gamma_s}} \frac{(1 + \gamma_s)^{-m_s}}{\Gamma(m_s + \frac{1}{2})} \frac{1}{2} F_{1} \left(1; m_s + \frac{1}{2}; m_s + 1; \frac{1}{1 + \gamma_s} \right), \]

\[ P_b = \frac{1}{2} \left[ 1 - \mu \sum_{k=0}^{m_s-1} \left( \frac{2k}{k} \right) \left( \frac{1 - \mu^2}{4} \right)^k \right], \mu = \frac{\gamma_s}{\sqrt{1 + \gamma_s}} \]
3. RESULTS & CONCLUSIONS

In this paper the concept of coherent DS-CDMA was understood and the BER expressions for the system were derived and verified. A RAKE receiver with perfect channel information was assumed here.

1. The multi path intensity profile is a very important design parameter for the DS-CDMA system. A small multi path intensity profile decaying will lead to the requirement of the paths with low average strengths are provided little information relative to the initial paths.

2. It was found that increasing average received SNR of the initial path much beyond 10dB only slightly improved the BER performance.

3. As per the graph between BER vs SNR for different values of ‘m’ in coherent DS-CDMA, as value of m increases bit error rate decreases.

4. As per the graph between BER vs SNR for different values of ‘$L_R$’ and ‘$L_p$’ in coherent DS-CDMA, as value of $L_R$ and $L_p$ increases bit error rate decreases.

5. The number of K users increase means BER value also increase.

RESULTS

![Graph 1](image1.png)  
Fig. 1 BER with parameters $m=1$, $K=50$, $L_R=1$, $\delta = 0.2$

![Graph 2](image2.png)  
Fig. 2 BER with parameters $m=0.75$, $\gamma_0=10$dB, $L_R=2$, $\delta = 0$

![Graph 3](image3.png)  
Fig. 3 BER with parameters $m=0.75$, $\gamma_0=10$dB, $L_p=10$, $\delta = 0.0$

![Graph 4](image4.png)  
Fig. 4 BER with parameters $m=0.75$, $\gamma_0=10$dB, $L_R=2$, $\delta = 0.2$
PERFORMANCE ANALYSIS

Fig. 5 BER with parameters $m=1$, $\gamma_0=10$dB, $L_R=L_p=1$

Fig. 6 BER with parameters $m=0.75$, $\delta=0$, $L_R=L_p=10$

REFERENCES


